

Covariant path integral formalism of relativistic quantum mechanics along proper time

H. Y. Geng¹

¹*Department of Quantum Engineering and Systems Science,
The University of Tokyo, Hongo 7-3-1, Tokyo 113-8656, Japan*

A space-time symmetric and explicitly Lorentz covariant path integral formalism of relativistic quantum mechanics is proposed, which produces partial locally correlations of quantum processes of massive particles with the velocity of light at low energy limits. A superluminal correlation is also possible if anti-particles that moving along reverse time direction are excited. This provides a new point of view to interpret EPR experiments, also leaks a light of hope for hidden variable theories.

PACS numbers: 03.65.-w, 31.15.Kb, 04.20.Fy, 03.30.+p, 03.65.Ud

Keywords: variational principles, path integral formalism, special relativity, partial locality

I. INTRODUCTION

In spite of quantum mechanics has demonstrated the amazing power to describe micro-phenomena, its behavior is so different from macro-intuitions that attempts to recover (quasi-) deterministic movement of particles never give up.^{1,2,3} However, due to non-local correlation of quantum states, these attempts all suffer quite difficulties to achieve self-consistent results and received a despaired strike by Aspect's experiments^{4,5} on EPR correlations.⁶ Many evidences have revealed that non-locality (or entanglement) is a fundamental property of quantum process, and to restore the motion of particles back to deterministic manner is in fact impossible.

Nevertheless, one may ask a further question: is such kind of non-local correlation globally (i.e., simultaneously around the whole space) correlated or partially localized? From a point of view of standard non-relativistic quantum mechanics, since wave function is defined on the whole space, as well as all operators, the theory must be globally correlated. In terms of Feynman's path integral formalism, that is to say that when calculating the transition function between two quantum states

$$\begin{aligned} \langle q_{t_b}, t_b | q_{t_a}, t_a \rangle &= \int_{q(t_a)=q_{t_a}}^{q(t_b)=q_{t_b}} \mathcal{D}[q(t)] \mathcal{D}[p(t)] \\ &\times \exp \left[\frac{i}{\hbar} \int_{t_a}^{t_b} dt \left\{ \sum_r p_r(t) \frac{d}{dt} q_r(t) - H(q(t), p(t)) \right\} \right], \end{aligned} \quad (1)$$

all paths across the space are allowed,^{7,8} which results in globally and instantaneous corrections. Here path integral $\int_{q(t_a)=q_{t_a}}^{q(t_b)=q_{t_b}} \mathcal{D}[q(t)] \mathcal{D}[p(t)]$ means to sum over the whole coordinate and momentum spaces along time-sliced paths with fixed end points at q_{t_a} and q_{t_b} (see chapter 1 of Ref. 8 for details). Thus one may wonder how can we separate an atomic (quantum) system from the universe and treat it isolatedly as we did heaps of times before? A consistent theory should explain why to ignore effects due to supernova explosions or other processes occurring at somewhere in the cosmos is reasonable. It is difficult within the framework of standard quantum mechanics¹⁸ Within the framework of path integral formalism, one may would like to employ the stationary phase condition to argue that contributions from those paths with large deviation from classical one's canceled completely and results in required isolation conditions. But notice (a) this is only true for quasi-classical approximations; (b) obviously different paths would have different variations of the contribution due to changes of the dynamic environment along each course, there is no guarantee that all of these variations with time can be completely and simultaneously canceled all the time. In a sense of that, we reach a point that some paths in Eq.(1) should be forbidden in a self-consistent quantum theory in order to satisfy the isolation hypothesis and to remove instantaneous correlations.

Actually, it will be seen in this paper that to generalize the Feynman's formalism slightly to covariant relativistic case will produce partial locality to quantum mechanics. This property is crucial to ensure that experiments on atomic system are really corresponding to the behavior of that system but not those relating to the external cosmos. It is necessary to point out that several attempts to set up a relativistic particle path integral have been proposed before, but they failed to produce the required isolation condition. The employed formalisms are rather unsatisfactory due to lack of explicit Lorentz covariance and treated time and spatial coordinates on different footings.^{9,10,11,12,13,14} The physical implications associated with the derivations also are not clear enough. Therefore these attempts are in fact a kind of mathematical techniques for conventional quantum mechanics, and cannot be taken as the foundation for developing a new theory. In subsequent sections, a variational principle on world lines is developed, which is necessary in order to generalize Feynman's formalism to Lorentz covariance. The relationship with standard quantum

mechanics is discussed in section III. Some understandings arised from this new point of view are given, followed by a comparison with previous derivations.

II. PRINCIPLE OF FORMAL ACTION

Thanks to the theory of relativity, time is deprived its privilege in motion equations and became an ordinary dimension of space-time. If one still would like to use Lagrangian variational principle to recover the position of action principle in physics, finding out an other parameter to take the place of time in Newtonian mechanics is necessary. Considering the movement of a particle with non-zero rest mass in space-time, we always can choose a frame of reference in which the particle is at rest. Usually it is non-inertial. However, it is reasonable to assume that instantaneously one can employ an inertial frame to approximate it (in terms of the general theory of relativity, the space-time manifold can be approximated by a series of Minkowski spacetimes locally¹⁵). In this way the movement of a particle can be described by a series of inertial frames, which relating to the observer by respective Lorentz transformations¹⁹

$$x_\mu(\beta) = \sum_\nu \Lambda_\mu{}^\nu(\beta) x_\nu^0. \quad (2)$$

Here β is the ratio of instantaneous velocity to light speed, i.e., v/c . Under these transformations, the length of proper time τ is invariant and satisfies $c^2(d\tau)^2 = \sum_{\mu,\nu} \eta^{\mu\nu} dx_\mu dx_\nu$, where $\eta^{\mu\nu}$ is the metric of Minkowski space. Usually τ can be expressed as a functional as $\tau = \tau(\beta, x_\mu)$. Thus the variation of frames characterized by parameter β can also be characterized by proper time τ . Namely, with Eq.(2) we have $x_\mu = x_\mu(\tau)$. In this way the proper time takes the privilege position of time as in Newtonian mechanics, and the motion equations of x_μ along τ can be obtained by the generalized principle of least action that defined on world lines.²⁰

Suppose a relativistic dynamical system that can be determined by a characteristic functional $L(x_\mu, \dot{x}_\mu)$, where $\dot{x}_\mu = dx_\mu/d\tau$, then one may have a formal action defined on world lines

$$S[x_\mu(\tau)] = \int_{\tau_i}^{\tau_f} L(x_\mu, \dot{x}_\mu) d\tau \quad (3)$$

with fixed end points at $x_\mu|_{\tau_i}$ and $x_\mu|_{\tau_f}$. The variation of this action with respect to world lines with fixed boundary conditions $\delta x_\mu(\tau_i) = \delta x_\mu|_{\tau_i} = 0$ and $\delta x_\mu(\tau_f) = \delta x_\mu|_{\tau_f} = 0$ gives

$$\frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{x}_\mu} \right) - \frac{\partial L}{\partial x_\mu} = 0 \quad (4)$$

when $\delta S[x_\mu(\tau)] = 0$. This is just the generalized covariant formalism of least action principle, and one can employ Eq.(4) to reproduce the covariant motion equation of particles.

Using the formal conjugate momentum $p^\mu = \partial L / \partial \dot{x}_\mu$, we can define a formal Hamiltonian by a Legendre transform

$$M(x_\mu, p^\mu) = \sum_\mu p^\mu \dot{x}_\mu - L(x_\mu, \dot{x}_\mu). \quad (5)$$

Usually it is Lorentz invariant. Then the equation of motion can also be given by

$$\dot{x}_\mu = \frac{\partial M}{\partial p^\mu}, \quad \dot{p}^\mu = -\frac{\partial M}{\partial x_\mu}. \quad (6)$$

It is evident from above discussion that if M is not explicitly τ -dependent, then the formal Hamiltonian is a conservative quantity. When x_μ and p^μ varied independently, the formal action of Eq.(3) can be expressed as

$$S[x_\mu(\tau), p^\mu(\tau)] = \int_{\tau_i}^{\tau_f} \left[\sum_\mu p^\mu \dot{x}_\mu - M(x_\mu, p^\mu) \right] d\tau, \quad (7)$$

which is Lorentz invariant too.

For a free particle, one has $dp^\mu/d\tau = 0$ for 4-momentum p^μ .¹⁵ It is easy to prove that the corresponding formal Hamiltonian should be

$$M = \frac{c}{2} \sqrt{p^\mu p_\mu}. \quad (8)$$

Then one gets $M = m_0 c^2 / 2$ where m_0 is the rest mass because $\sum_\mu p^\mu p_\mu = m_0^2 c^2$. The conservation law of M in fact becomes the conservation of rest mass or rest energy. In this formalism we see that mass no longer appears as an *a priori* parameter but a dynamical variable analogous to Hamiltonian in classical physics.

III. COVARIANT PATH INTEGRAL

As discussed in introduction section, self-consistent quantum mechanics should be at least partial localized. If it is true, there is no any reason to save the conception that wave function (or quantum state) exclusively describes just the motion of that particles.²¹ This clew of thought eventually leads to a theory of quantum fields. However, here we would like to restrain our discussions within quantum mechanics.

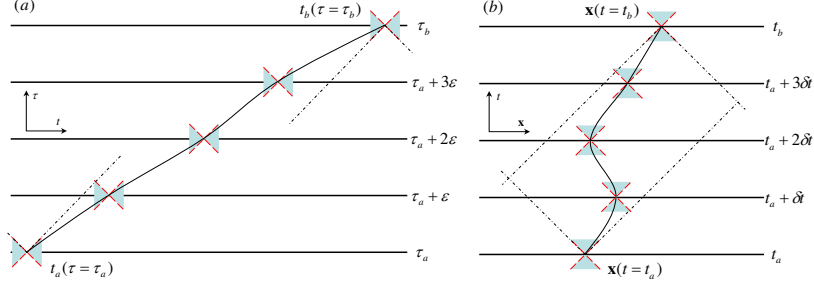


FIG. 1: Projections of allowed paths in covariant path integral formalism in (1+1) space-time. (a) proper time slices for Eq.(9) where paths must satisfy $|d\tau/dt| \leq 1$ (domain \mathbb{C}_1); (b) spatial summation lattice between time t_a and t_b (domain \mathbb{C}_2), not only all paths belonging to end points $\mathbf{x}(t_a)$ and $\mathbf{x}(t_b)$ are restricted by the dot-dashed rectangle, every path also should be within the instantaneous light cones all the time to ensure a properly definition of proper time.

Following Feynman's consideration, it is quite natural to assume that the transition probability amplitude between two quantum states is completely determined by a phase factor given by the formal action of particle Eq.(7), which results in the covariant version of Eq.(1)

$$\begin{aligned} \langle x_\mu(\tau_b), \tau_b | x_\mu(\tau_a), \tau_a \rangle &= \int_{x_\mu(\tau_a)=x_\mu|_{\tau_a}}^{x_\mu(\tau_b)=x_\mu|_{\tau_b}} \mathcal{D}[x(\tau)] \mathcal{D}[p(\tau)] \\ &\times \exp \left[\frac{i}{\hbar} \int_{\tau_a}^{\tau_b} d\tau \left\{ \sum_{\mu, \nu} p^\nu(\tau) \frac{d}{d\tau} x_\mu(\tau) \delta_{\mu\nu} - M(x_\mu(\tau), p^\mu(\tau)) \right\} \right]. \end{aligned} \quad (9)$$

Evidently, this is just a natural special relativity generalization of Feynman's path integral formalism. The proper time sliced expression is given by (differs from other path integral formalisms where a time-slicing scheme is used)

$$\begin{aligned} \langle x_\mu(\tau_b), \tau_b | x_\mu(\tau_a), \tau_a \rangle &= \lim_{\substack{n \rightarrow \infty \\ n \cdot \delta\tau = \tau_b - \tau_a}} \int_{\mathbb{C}} \prod_{k=1}^{n-1} \frac{d^4 x_{\tau_k}}{(2\pi\hbar)^2} \int \prod_{k=0}^{n-1} \frac{d^4 p_{\tau_k}}{(2\pi\hbar)^2} \\ &\times \exp \left[\frac{i}{\hbar} \sum_{k=0}^{n-1} \left\{ \sum_{\mu, \nu} p^\nu(\tau_k) \delta x_\mu(\tau_k) \delta_{\mu\nu} - \delta\tau \cdot M(x_\mu(\tau_k), p^\mu(\tau_k)) \right\} \right], \end{aligned} \quad (10)$$

where $\delta x_\mu(\tau_k) = x_\mu(\tau_{k+1}) - x_\mu(\tau_k)$ and the integral domain \mathbb{C} of space-time coordinates is not only determined by boundary conditions $x_\mu(\tau_b) = x_\mu|_{\tau_b}$ and $x_\mu(\tau_a) = x_\mu|_{\tau_a}$, it also has to satisfy a condition of that $\delta\tau$ should be defined properly which constrained by physical conditions $|d\tau/dt| \leq 1$ and $\sum_{\mu, \nu} \eta^{\mu\nu} dx_\mu dx_\nu \geq 0$. Thus we have $\mathbb{C} = \mathbb{C}_1 \cup \mathbb{C}_2$, corresponding to an intrinsic property of Minkowski space-time and the non-spatial like motions of particle, respectively. It is clear that the most significant difference between this and the conventional path integral formalism lies on that in the latter case, all paths across the space are possible, while Eq.(9) allows just some special paths, as showed in Fig.1. For simplicity, only (1+1)-dimension case is shown here, where time-component of proper time sliced paths are restricted by $|d\tau/dt| \leq 1$ and spatial component belonging to end points $\mathbf{x}(t_a)$ and $\mathbf{x}(t_b)$ are not only limited within the dot-dashed rectangle (relativistic causality), but also each path should be time-like, i.e., contained in the instantaneous light cones all the time, to guarantee $(d\tau)^2 \geq 0$ at classical limit (see Eq.(9)). Evidently, this property recovers the global causality of relativity and partial locality to the theory, the latter eventually leads to the isolation condition for massive particles. It is easy to see that Eq.(9) contains conventional quantum mechanics as approximation as Feynman case shown. The superposition and composition law of the transition probability amplitude is stated in a form of

$$\int_{x_\mu(\tau_a)}^{x_\mu(\tau_b)} \mathcal{D}[x] \mathcal{D}[p] = \int_{\mathbb{C}} d^4 x' \int_{x'_\mu(\tau')}^{x_\mu(\tau_b)} \mathcal{D}[x_{II}] \mathcal{D}[p_{II}] \int_{x_\mu(\tau_a)}^{x'_\mu(\tau')} \mathcal{D}[x_I] \mathcal{D}[p_I]. \quad (11)$$

It is worthwhile to note that J. Schwinger once introduced proper time depended Green functions to trick the gauge invariant problem,¹⁶ which is different from our considerations here completely.

Consider the transition function of Eq.(9) with an infinitesimal increase in proper time τ , the wave function can be simplified as

$$\begin{aligned} \psi(x_\mu, \tau_0 + \varepsilon) &= \int_{\mathbb{C}} d^4 x^0 \int \frac{d^4 p}{(2\pi\hbar)^4} \psi(x_\mu^0, \tau_0) \\ &\times \exp \left[\frac{i}{\hbar} \left\{ \sum_{\mu, \nu} p^\nu (x_\mu - x_\mu^0) \delta_{\mu\nu} - \varepsilon \cdot M(x_\mu, p^\mu) \right\} \right], \end{aligned} \quad (12)$$

with $\psi(x_\mu) = \langle x_\mu | \psi \rangle$. As matrix element of operator $O(\hat{x}_\mu(\tau))$ is given by⁸

$$\langle x_{\mu|\tau_b} | O(\hat{x}_\mu(\tau)) | x_{\mu|\tau_a} \rangle = \int_{\mathbb{C}} \mathcal{D}[x(\tau)] \mathcal{D}[p(\tau)] O(x_\mu(\tau)) \exp \left[\frac{i}{\hbar} S(\tau_b, \tau_a) \right], \quad (13)$$

it is quite easy to prove that the corresponding form of operators acting on wave function Eq.(12) are

$$i\hbar \frac{\partial}{\partial \tau} \Leftrightarrow \hat{M}, \quad -i\hbar \frac{\partial}{\partial x_\mu} \Leftrightarrow \hat{p}^\mu \quad (14)$$

to the first order of ε when ε approaches zero. Note here that the 4-momentum operators have an opposite sign comparing to traditional definition, but it does not matter. Using Eq.(14) we have the motion equation for wave function as

$$i\hbar \frac{\partial}{\partial \tau} \Psi = \hat{M} \Psi. \quad (15)$$

For a formal Hamiltonian eigenstate $i\hbar \partial \Psi / \partial \tau = m \Psi$, one gets

$$\hat{M} (\partial / \partial x_\mu) \psi = m \psi, \quad (16)$$

with $\Psi = \psi \exp(-im\tau/\hbar)$.

If studied system only includes gauge interactions, then the formal Hamiltonian of particle part still keeps the form of Eq.(8) except for a slight modification $p^\mu \rightarrow p^\mu + A^\mu$. So generally we can use the form of Eq.(8) for discussion. Applying one more \hat{M} on Eq.(16) and making use of Eqs.(8,14) and $m = m_0 c^2 / 2$, we reach the Klein-Gordon equation. A detailed prove is given in appendix A to show this formal inference is true. In order to obtain a linear equation, let's consider a mapping from 4-vector to a Lorentz scalar $x_\mu \rightarrow x = \sum_\mu \gamma^\mu x_\mu$. Conservation of the length requires $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ with anticommutation defined by $\{A, B\} = AB + BA$. Obviously γ^μ are exactly the Dirac matrices. In this sense $\gamma^\mu / \sqrt{2}$ form a set of orthonormal basis with inner product given by anticommutation operation, and 4-vector x_μ can be viewed as the components of Lorentz scalar x on this basis set $x_\mu = \{x, \gamma_\mu\} / 2$. Then the formal action can be reexpressed as

$$S[x(\tau), p(\tau)] = \int_{\tau_i}^{\tau_f} \left[\frac{1}{2} \{p, \dot{x}\} - M(x, p) \right] d\tau, \quad (17)$$

with M in Eq.(8) given by $M = pc/2 = \sum_\mu \gamma_\mu p^\mu c/2$, which is a Lorentz scalar but usually is a matrix too. Then the Dirac equation is obtained directly from Eq.(16). A derivation of non-relativistic approximation is given in appendix B, which reproduced the standard non-relativistic path internal formalism for free particles, demonstrating the validity of our proposal.

By comparing with previous attempts to the same issue, our proposed scheme is more systematic and elegant with high space-time symmetry. Though all proposed methods have ability to reproduce relativistic equation of motion, the works of D. Felea⁹ and P. Gosselin¹² are not Lorentz covariant explicitly. Without introducing proper time, they treated time and spatial coordinates non-equivalently, which is contrary to the spirit of relativity. T. Padmanabhan¹³ employed a quite different approach and introduced proper time, unfortunately it is still non-covariant. In Pazma's method even the action is non-invariant.¹⁰ The only covariant version is originated from Feynman, but it suffers a difficulty that the physical meanings of the procedure and the conjugated variable of proper time are unclear. In a sense of that, these path integral schemes are actually just convenient mathematical techniques for quantum mechanics, and cannot be treated as the basis for developing a new theory. By contrast, our proposed formalism is not only Lorentz covariant, but also bears clear physical implications by developing the rest energy operator \hat{M} , which results in the evolution of states controlled by \hat{M} besides the conventional Hamiltonian, therefore has the potential to treat variable rest mass problem. Moreover, all previous proposed formalisms are just defined on coordinate space, It is well known that this kind of path integral is not always valid, and including phase-space integrals, as in our scheme, is more appropriate.⁸

IV. DISCUSSION

It would be interesting to show how our formalism can produce the partial locality for quantum process, for example, measurements, which is beyond the scope of application of other proposals. For simplicity, let's assume the measurements are just perturbations to the state, and the corresponding action can be characterized by a Dirac delta function as $\alpha S' \delta(x_\mu - x'_\mu)$. Then the wave function with two measurements performed respectively at space-like points of x'_u and x''_u is given by

$$\psi(x_\mu, \tau) = \int_{\mathbb{C}} \psi(x_\mu^0, \tau_0) \mathcal{D}[x] \exp\left[\frac{i}{\hbar} \{S_0 + \alpha S' \delta(x_u - x'_\mu) + \beta S'' \delta(x_u - x''_\mu)\}\right]. \quad (18)$$

Here we have suppressed unrelated expressions for brief, for example the path integral in phase space and the detailed information about actions. Parameters α and β must be small quantities to ensure measurements are perturbation. Making Taylor expansion of the exponential to first order of α and β , we get

$$\psi(x_\mu, \tau) \simeq \psi_0(x_\mu, \tau) + \psi'_{D_1}(\alpha) + \psi''_{D_2}(\beta). \quad (19)$$

The first term at the right hand side is the wave function without perturbations and the last two terms corresponding to influences of measurements whose subscripts D_1 and D_2 indicate the domains of definition, which given by

$$\psi'_{D_1}(\alpha) = \frac{i}{\hbar} \int_{\mathbb{C}} \mathcal{D}[x] \psi(x_\mu^0, \tau_0) \alpha S' \delta(x_u - x'_\mu) \exp\left[\frac{i}{\hbar} S_0\right]. \quad (20)$$

Similar expression holds for $\psi''_{D_2}(\beta)$. It is interesting to notice that the paths with non-zero contributions to Eq.(20) must pass through point x'_μ . On the other hand, the integral domain \mathbb{C} is determined by non-spatial condition and $|d\tau/dt| \leq 1$ condition, the latter allows us to divide the integral as

$$\int_{\mathbb{C}} \mathcal{D}[x] = \int_{\mathbb{C}'} \mathcal{D}[x] + \int_{\mathbb{C}''} \mathcal{D}[x] \quad (21)$$

where \mathbb{C}' only contains paths with $d\tau/dt \leq 1$ and other paths belong to \mathbb{C}'' . Evidently, the paths of \mathbb{C}'' must contain contributions arised from $d\tau/dt \geq -1$, which describes a particle moving along reverse time, that is to say, it corresponds to an anti-particle.

At low energy approximation where no anti-particles are excited, we can ignore the paths belonging to \mathbb{C}'' safely. In this case, at time (t, τ) , the value of wave function are determined by integral over those paths belonging to \mathbb{C}' and passing trough x'_μ which constrained by non-spatial condition $|\vec{x}| \leq c(t - t')$, see Fig.1. This means that $\psi'_{D_1}(\alpha)$ does not be defined around the whole space-time. To enlarge its definition domain to the whole space-time, we have to use a step function $H(x)$ that has a value of 1 if $x \geq 0$ otherwise 0 to characterize the definition domain D_1 explicitly. The result reads

$$\psi'_\alpha(x_\mu, \tau) = H(\Sigma_\mu \Delta x_\mu \Delta x^\mu) H(1 - d\tau/dt) \psi'_{D_1}(\alpha), \quad (22)$$

where $\Delta x_\mu = x_\mu - x'_\mu$. Similar expression holds for $\psi''_{D_2}(\beta)$. Thus if define $t_c = |\vec{x}' - \vec{x}''|/2c + (t' + t'')/2$, then when $t \leq t_c$ one gets

$$\begin{aligned} \int \psi'^*_\alpha(\vec{x}, t) \psi''_\beta(\vec{x}, t) d\vec{x} &= \int_{D_1 \cap D_2} \psi'^*_{D_1}(\alpha) \psi''_{D_2}(\beta) d\vec{x} \\ &= \int_{\emptyset} \psi'^*_{D_1}(\alpha) \psi''_{D_2}(\beta) d\vec{x} \equiv 0. \end{aligned} \quad (23)$$

That is, the influence of two measurements occurred at x'_μ and x''_μ are physically uncorrelated. However, when $t > t_c$, it will become correlated since the intersection of D_1 and D_2 usually non-empty in this case.

It is quite clear that in this case the propagation of quantum correlation of measurements also has the same speed as light (if the space-time coordinates of one measurement lies within the influence region of another measurement, then their results will be correlated, otherwise are independent). However, if paths in \mathbb{C}'' are involved, i.e., with anti-particle excitations, the segments of a path with $d\tau/dt \geq -1$ allow the path sweeping a larger time interval than actually happened from the point of an observer. This implies that the permitted spatial region in Fig.1(b) are broadened, which would lead to a superluminal propagation of influence region of measurements. At high energy limits where excited anti-particle can exist permanently, quantum correlation then will approach globally and instantaneously.

This novel mechanism for propagation of quantum correlation makes it possible to interpret the experiments on EPR effect⁶ with a new point of view. Aspect's experiments shows a contradiction between the reality and locality in a physical theory.^{4,5} However since there the locality is defined by the principle of maximum speed of light (i.e., the velocity of quantum correlation propagation is limited by c), the propagation mechanism of perturbations arised from our formalism releases this definition and solve the contradiction automatically for massive matters. As long as energy does not approach infinite, the life-time of excited anti-particles must be finite, then we have a finite correlation speed and the required isolation condition for quantum mechanics. If we define this property as locality, which has the same spirit as the original one—to get rid of action at a distance from physical theory, then Bell's inequality¹⁷ loses its availability to characterize the relationship of physical reality and locality because we don't know exact information about anti-particle excitations, as well as the exact value of correction speed, despite it should larger than the velocity of light usually. we see that in this case we still have a possibility to preserve the reality and locality from EPR's experiments. Of course this also leaks a light of hope for hidden variable theories if they prefer to include anti-particle effects in their framework to reproduce superluminal mechanism, though I do not believe it could be a right choice for the truth. It is necessary to point out that in Aspect's experiments, since correlation between photons are examined, which corresponding to a singular case in our formalism because photon is massless and its anti-particle is just itself, the predicted correlation velocity will approach infinite. This should be the only one case where action at a distance may be possible in our formalism and results in a maximum violation of Bell's inequality. However, to eliminate this possibility is far beyond the scope of this paper. Regards this, we suggest to perform analogous experiments on massive particles to examine the effects of anti-particle excitations.

V. CONCLUSION

Using developed variational principle defined on world lines, we obtained the formal action for a relativistic system. It permits us to generalize the Feynman's path integral formalism of quantum mechanics to special relativity case elegantly, which reproduces the Klein-Gordon and Dirac equation for infinitesimal intervals very well. Some novel understandings are obtained from this new point of view. Though quantum state can be defined on the whole space-time, quantum processes are always localized at low energy limits, while large-scale correlations are due to anti-particles excitations, which makes superluminal corrections become possible even within the framework of special relativity. With this mechanism for correlation propagation, quantum mechanics becomes more completeness and self-consistency. Also it shows at the first time that we still have opportunity to save the concept of physical reality without lost the locality of a theory. Sometimes this kind of correlation mechanism will bring a little inconvenience to calculation. For example, when considering a process described by a superposition state, one then need to check that if its sub-states are really correlated or not within studied time interval in order to give physical meaning to the final superimposed state. At last, it is necessary to note that most discussions in this paper are for single particle, but the same conclusions hold for many-particles case, which is rather straightforward to derive and don't need to repeat here.

Acknowledgments

The author thankfully acknowledges the help and kind hospitality from Prof. Ying Chen at the University of Tokyo.

APPENDIX A: KLEIN-GORDON EQUATION

Here we would like to derive the Klein-Gordon equation of free particle explicitly by starting from Eq.(12), which can be rewritten as

$$\begin{aligned} \psi(x_\mu, \tau + \varepsilon) = & \int_{\mathbb{C}} d^4x^0 \int \frac{d^4p}{(2\pi\hbar)^4} \psi(x_\mu^0, \tau) \\ & \times \exp \left[\frac{i}{\hbar} \left\{ \sum_{\mu, \nu} p^\nu (x_\mu - x_\mu^0) \delta_{\mu\nu} - \varepsilon \cdot \sum_{\nu} \frac{p^\nu p_\nu}{2m_0} \right\} \right], \end{aligned} \quad (\text{A1})$$

here we have used a condition of $\sum_{\mu} p_\mu p^\mu = m_0^2 c^2$ to remove the square root in action, which means that we are considering a particle with fixed rest mass. Note the 4-momentum in Eq.(A1) should be treated independently when

making path integral, namely, it does not satisfy the mass-energy relation and corresponding to none real physical state. Integral out the momentum, one gets

$$\psi(x_\mu, \tau + \varepsilon) = \frac{im_0^2}{4\pi^2\hbar^2\varepsilon^2} \int_{\mathbb{C}} d^4(\delta x) \psi(x_\mu - \delta x_\mu, \tau) \exp \left[\frac{i}{\hbar} \frac{m_0}{2\varepsilon} \sum_{\mu} \delta x_\mu \delta x^\mu \right]. \quad (\text{A2})$$

Here use has being made of $\delta x_\mu = x_\mu - x_\mu^0$. As δx_μ is small and making the Taylor expansion of ψ to its second order, we have

$$\psi(x_\mu^0) \simeq \psi(x_\mu) - \sum_{\mu} \delta x_\mu \cdot \frac{\partial \psi}{\partial x_\mu} + \frac{1}{2!} \sum_{\mu, \nu} \delta x_\mu \delta x_\nu \cdot \frac{\partial^2 \psi}{\partial x_\mu \partial x_\nu} + \dots \quad (\text{A3})$$

While considering that the integral domain \mathbb{C} is central symmetry with respect to the origin of coordinates, non-zero contributions to the integral are just come from terms of $\psi(x_\mu) + \frac{1}{2} \sum_{\mu} (\delta x_\mu)^2 \cdot \frac{\partial^2 \psi}{\partial x_\mu^2}$. Thus we can write the right hand side of Eq.(A2) as a sum of two terms, one depends on ψ and the other depends on second order derivatives of ψ .

Let's calculate the first term firstly, i.e.,

$$\begin{aligned} F.t. &= \frac{im_0^2}{4\pi^2\hbar^2\varepsilon^2} \int_{\mathbb{C}} d^4(\delta x) \psi(x_\mu, \tau) \exp \left[\frac{i}{\hbar} \frac{m_0}{2\varepsilon} \sum_{\mu} \delta x_\mu \delta x^\mu \right] \\ &= \frac{im_0^2}{4\pi^2\hbar^2\varepsilon^2} \psi(x_\mu, \tau) \int_{\mathbb{C}_1} d(c\delta t) \int_{\mathbb{C}_2} d^3(\delta \vec{x}) \exp \left[\frac{i}{\hbar} \frac{m_0}{2\varepsilon} \sum_{\mu} \delta x_\mu \delta x^\mu \right] \end{aligned} \quad (\text{A4})$$

where \mathbb{C}_1 is given by $[-\infty, -c\varepsilon] \cup [c\varepsilon, \infty]$ and \mathbb{C}_2 is determined by $|\delta \vec{x}| \leq c\delta t$ (see Fig.1). Define $\alpha = \frac{m_0}{2\varepsilon\hbar}$ and integral out the spatial components, we have

$$F.t. = \frac{im_0^2}{4\pi^2\hbar^2\varepsilon^2} \psi(x_\mu, \tau) \int_{\mathbb{C}_1} \frac{i\pi}{\alpha} e^{-i\alpha(c\delta t)^2} \sqrt{\frac{\pi}{i\alpha}} (e^{i\alpha(c\delta t)^2} - 1) d(c\delta t). \quad (\text{A5})$$

Taking account of $e^{i\alpha(c\delta t)^2}$ approaches zero when $\varepsilon \rightarrow 0$, we can make an approximation of the square root simply as $\sqrt{i\pi/\alpha}$ to get accurate enough result on $O(\varepsilon)$. Thus integral over the time component and making use of $e^{\pm i\alpha\infty^2} \rightarrow 0$, we finally get

$$\begin{aligned} F.t. &= \frac{-m_0^2}{4\pi^2\hbar^2\varepsilon^2} \psi(x_\mu, \tau) \left(\frac{i\pi}{\alpha} \right)^2 e^{-i\alpha(c\varepsilon)^2/2} \\ &= \psi(x_\mu, \tau) e^{-i\alpha(c\varepsilon)^2/2} \simeq \psi(x_\mu, \tau) - \frac{im_0c^2}{4\hbar} \varepsilon \cdot \psi(x_\mu, \tau). \end{aligned} \quad (\text{A6})$$

In the last equality we just keep to the first order of ε .

The calculation of the second term are quite similar,

$$\begin{aligned} S.t. &= \frac{im_0^2}{8\pi^2\hbar^2\varepsilon^2} \int_{\mathbb{C}} d^4(\delta x) \left[\sum_{\mu} (\delta x_\mu)^2 \cdot \frac{\partial^2 \psi}{\partial x_\mu^2} \right] \exp \left[i\alpha \sum_{\mu} \delta x_\mu \delta x^\mu \right] \\ &= \frac{im_0^2}{8\pi^2\hbar^2\varepsilon^2} \left[\sum_{\mu} \partial^\mu \partial_\mu \psi \right] \left(\frac{i\pi}{\alpha} \right)^{\frac{3}{2}} \int_{\mathbb{C}_1} (c\delta t)^2 e^{-i\alpha(c\delta t)^2} d(c\delta t). \end{aligned} \quad (\text{A7})$$

Integral term gives

$$\frac{c\varepsilon}{i\alpha} e^{-i\alpha(c\varepsilon)^2} + \frac{1}{i2\alpha} \sqrt{\frac{\pi}{i\alpha}} e^{-i\alpha(c\varepsilon)^2/2}. \quad (\text{A8})$$

Regardless the exponential contributions, the first term is in proportion to $O(\varepsilon^2)$ and the second term in proportion to $O(\varepsilon^{3/2})$. At the first approximation, we can neglect the first term. Then we have

$$\begin{aligned} S.t. &\simeq \frac{im_0^2}{8\pi^2\hbar^2\varepsilon^2} \left[\sum_{\mu} \partial^\mu \partial_\mu \psi \right] \left(\frac{i\pi}{\alpha} \right)^{\frac{3}{2}} \frac{1}{i2\alpha} \sqrt{\frac{\pi}{i\alpha}} e^{-i\alpha(c\varepsilon)^2/2} \\ &= \frac{i}{4\alpha} e^{-i\alpha(c\varepsilon)^2/2} \sum_{\mu} \partial^\mu \partial_\mu \psi \simeq \frac{i\hbar\varepsilon}{2m_0} \sum_{\mu} \partial^\mu \partial_\mu \psi. \end{aligned} \quad (\text{A9})$$

In this way we get

$$\psi(x_\mu, \tau + \varepsilon) = \psi(x_\mu, \tau) - \frac{im_0c^2}{4\hbar}\varepsilon \cdot \psi(x_\mu, \tau) + \frac{i\hbar\varepsilon}{2m_0} \sum_\mu \partial^\mu \partial_\mu \psi(x_\mu, \tau). \quad (\text{A10})$$

Expand the left hand term to first order of ε and comparing with right hand terms, one has one trivial identical equality and

$$\frac{\partial \psi}{\partial \tau} + \frac{im_0c^2}{4\hbar}\psi = \frac{i\hbar}{2m_0} \sum_\mu \partial^\mu \partial_\mu \psi. \quad (\text{A11})$$

for first order of ε . Define $\phi = e^{-\frac{im_0c^2}{4\hbar}\tau}\psi$, one can rewrite above equation as

$$\frac{\partial \phi}{\partial \tau} = \frac{i\hbar}{2m_0} \sum_\mu \partial^\mu \partial_\mu \phi, \quad (\text{A12})$$

or,

$$-\hbar^2 \frac{\partial^2 \phi}{\partial \tau^2} = -\frac{\hbar^2 c^2}{4} \sum_\mu \partial^\mu \partial_\mu \phi, \quad (\text{A13})$$

i.e., $-\hbar^2 \partial^2 \phi / \partial \tau^2 = \hat{M}^2 \phi$ with $\phi = \phi(x_\mu) e^{-\frac{im_0c^2}{2\hbar}\tau}$. For space-time component $\phi(x_\mu)$, it satisfies

$$m_0^2 c^2 \phi(x_\mu) = -\hbar^2 \sum_\mu \partial^\mu \partial_\mu \phi(x_\mu), \quad (\text{A14})$$

which is exactly the Klein-Gordon equation.

APPENDIX B: NON-RELATIVISTIC APPROXIMATION

To obtain the non-relativistic approximation, it is natural to set light speed $c \rightarrow \infty$. Then we have $d\tau/dt \rightarrow 1$ at classical limit for massive particles when time-arrow (i.e., the direction of proper time is identical with time) is employed. This indicates that the domain \mathbb{C}_1 in figure 1(a) becomes a straight line which is given by $\tau = t_0 + t$, and the proper time is dependent and superfluous. The summation lattice in figure 1(b) also becomes equivalent to the standard path integral of non-relativistic quantum mechanics because $|\vec{x}| \leq c \cdot \delta t \rightarrow \infty$.

Under this approximation, the action can be expressed as

$$\int_{\tau_i}^{\tau_f} L(x_\mu, \dot{x}_\mu) d\tau \rightarrow \int_{t_i}^{t_f} L(\vec{x}, \frac{d\vec{x}}{dt}) dt + const., \quad (\text{B1})$$

namely, the non-relativistic action between t_i and t_f plus a constant because $\int d\tau = \int dt$ and $dx_\mu/d\tau = (dt/d\tau) \cdot dx_\mu/dt = dx_\mu/dt$. Take a free particle as example, the Lagrangian is

$$L = \frac{m_0}{2} \sum_\mu \dot{x}_\mu \dot{x}_\mu \rightarrow \frac{m_0 c^2}{2} - \frac{m_0 \vec{v}^2}{2} = \frac{m_0 c^2}{2} + L_{nr} \quad (\text{B2})$$

where subscript nr refers to standard non-relativistic Lagrangian. Then the action can be written as $S \rightarrow (t_f - t_i)m_0c^2/2 + S_{nr}$. Therefore from Eq.(9) we have the transition probability amplitude between two quantum states as

$$\begin{aligned} \langle \vec{x}, t_f | \vec{x}, t_i \rangle &= \lim_{n \rightarrow \infty} \prod_{n \cdot \delta t = t_f - t_i} \int_{t_i}^{t_f} \prod_{k=1}^{n-1} \frac{dt_k}{(2\pi\hbar)^{1/2}} \prod_{k=0}^{n-1} \frac{dE_k}{(2\pi\hbar)^{1/2}} \\ &\times \int_{x(t)=x(t_i)}^{x(t)=x(t_f)} \prod_{k=1}^{n-1} \frac{d^3x_{t_k}}{(2\pi\hbar)^{3/2}} \int \prod_{k=0}^{n-1} \frac{d^3p_{t_k}}{(2\pi\hbar)^{3/2}} \cdot e^{\frac{i}{\hbar}S_{nr}} \cdot e^{\frac{i}{\hbar}\frac{m_0c^2}{2}(t_f-t_i)}. \end{aligned} \quad (\text{B3})$$

Taking out the constant $v^n e^{\frac{i}{\hbar} \frac{m_0 c^2}{2} (t_f - t_i)}$ where

$$v^n = \lim_{\substack{n \rightarrow \infty \\ n \cdot \delta t = t_f - t_i}} (\delta t)^{n-1} \prod_{k=0}^{n-1} \int \frac{dE_k}{2\pi\hbar}, \quad (\text{B4})$$

we get the transition function as

$$\langle \vec{x}, t_f | \vec{x}, t_i \rangle \propto \lim_{\substack{n \rightarrow \infty \\ n \cdot \delta t = t_f - t_i}} \int_{x(t)=x(t_i)}^{x(t)=x(t_f)} \prod_{k=1}^{n-1} \frac{d^3 x_{t_k}}{(2\pi\hbar)^{3/2}} \int \prod_{k=0}^{n-1} \frac{d^3 p_{t_k}}{(2\pi\hbar)^{3/2}} \cdot e^{\frac{i}{\hbar} S_{nr}}. \quad (\text{B5})$$

The right side is exactly the standard non-relativistic path integral formalism for free particles.⁸ When potentials due to pure gauge fields are present, the conclusion still holds where instead Eq.(7) should be used.

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- ¹⁸ In wave dynamics, since wave functions are defined on the whole space, when considering an atomic system, say, hydrogen atom, then the Schrödinger equation should also include terms corresponding to other ions distributing across the universe (even charged black holes and strong magnetic fields). many of these ions have much larger charges than proton, as well as larger contributions to system energy. In this situation, the effects due to proton can be neglected, and one cannot obtain the well-known quantum properties of hydrogen atom, which has been confirmed by experiments many times. To avoid this difficulty, one has to assume that electron is restricted locally within some spatial region and satisfies certain isolation assumption. But it is incompatible with the postulations of standard quantum mechanics, as well as the boundary conditions.
- ¹⁹ If all interactions are instantaneous (as well recognised), non-inertial reference frame loses its intrinsic physical meaning and this quasi-inertial approximation becomes exact.
- ²⁰ For massless particles, Eq.(2) fails and the equations of motion is independent from proper time. It corresponds to a static case in Newtonian framework.
- ²¹ Partial localization inevitably leads to a concept of correlation length between quantum processes. Owing to the law of causality, measurements beyond correlation length on the same state should be independent, which will result in logical contradiction if one insists, say, single particle wave function just describes the motion of only *that* particle, because independent measurements will give an odd conclusion such that a particle may appear at different places in spatial space simultaneously.